day 19

A rational function is a function of the form $f(x) = \frac{p(x)}{q(x)}$,

where $p(x)$ and $q(x)$ are polynomials.

[$q(x)$ obviously not 0 everywhere]

End behavior

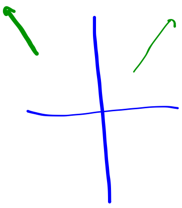
* 1 horizontal asymptote

$$\lim_{x \to +\infty} f(x) = \lim_{x \to -\infty} f(x) = L$$

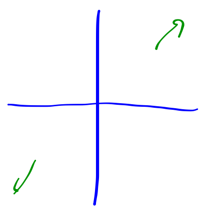
* $\longmapsto \infty$

* $\longmapsto -\infty$

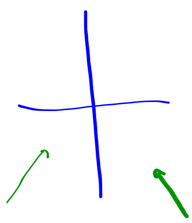
End Behavior of Power Functions



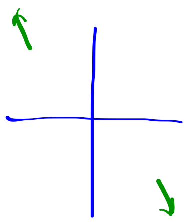
if degree even ≥ 2
$a > 0$



if degree odd ≥ 1
$a > 0$



if degree even ≥ 2
$a < 0$



if degree odd ≥ 1
$a < 0$

a is leading coefficient

day 19

Real Numbers

algebraic numbers
root of a polynomial equation w/rational coefficients

- rational #s
  ratio of 2 integers
  - → integers
    whole numbers negatives & whole #s.
  - → natural numbers
    1, 2, 3, . . .
  - → whole numbers
    ~~whole~~ natural numbers plus 0
- irrational #s
  $\sqrt{\text{non perfect square}}$

transcendental numbers
NOT the root of ANY polynomial equation with rational coefficients

π

e

[[ALL irrational]]

algebraic function:
"plug it into a polynomial w/ rational coefficients"
and it works

transcendental function

CAN'T

$2^x, 3^x, e^x$

day 19

1) $\lim_{x \to 0} \frac{\sin x}{x^2} =$

2) $\lim_{x \to 0} \frac{\sin(3x)}{x} =$

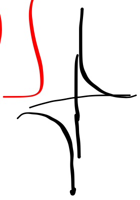
3) $f(x) = 3x^2$ use a limit definition of the derivative to find $f'(x)$

Use what you found to write the equation of the line tangent to the curve of *f*
at $x = 3$

Sep 23-6:33 PM

day 19

1) $\lim_{x\to 0}\frac{\sin x}{x^2} = \lim_{x\to 0}\left[\left(\frac{\sin x}{x}\right)\left(\frac{1}{x}\right)\right] = \text{DNE}$



Recall $\lim_{x\to 0}\frac{\sin x}{x} = 1$

2) $\lim_{x\to 0}\frac{\sin(3x)}{x} = \lim_{x\to 0} 3\left(\frac{\sin(3x)}{(3x)}\right) = 3$

3) $f(x) = 3x^2$ use a limit definition of the derivative to find $f'(x)$

Use what you found to write the equation of the line tangent to the curve of f at $x = 3$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{3(x+h)^2-3x^2}{h}$$
$$= \lim_{h\to 0}\frac{3(x^2+2xh+h^2)-3x^2}{h}$$
$$= \lim_{h\to 0}\frac{3x^2+6xh+3h^2-3x^2}{h}$$
$$= \lim_{h\to 0}\frac{6xh+3h^2}{h} = \lim_{h\to 0}(6x+3h) = 6x$$

$$f'(x) = \lim_{w\to x}\frac{f(w)-f(x)}{w-x}$$
$$= \lim_{w\to x}\frac{3w^2-3x^2}{w-x}$$
$$= \lim_{w\to x}\frac{3(w+x)(w-x)}{(w-x)}$$
$$= \lim_{w\to x} 3(w+x) = 3(x+x)$$
$$= 3(2x)$$
$$= 6x$$

slope of tan line $= f'(3) = 6(3) = 18$

pt: $(3, f(3)) = (3, 3(3)^2) = (3, 27)$

$y - 27 = 18(x-3)$

day 19

$f(3)$ is a NUMBER

$f(x)$ is a function

$f'(3)$ is a derivative, number, slope of the tangent line at $x=3$, instantaneous rate of chg of $f(x)$ at $x=3$

$f'(x)$ is a derivative, function

$\boxed{f'(x)}'$ is a derivative function

* second derivative
* $f''(x)$, $f^{(2)}(x)$
* $\frac{d^2y}{dx^2}$ "take two derivatives of y, with respect to x BOTH times"

$f'''(x) = f^{(3)}(x)$ . . .

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day 19

$$A(x,y) = x+y$$

$$S(x,y) = x-y$$

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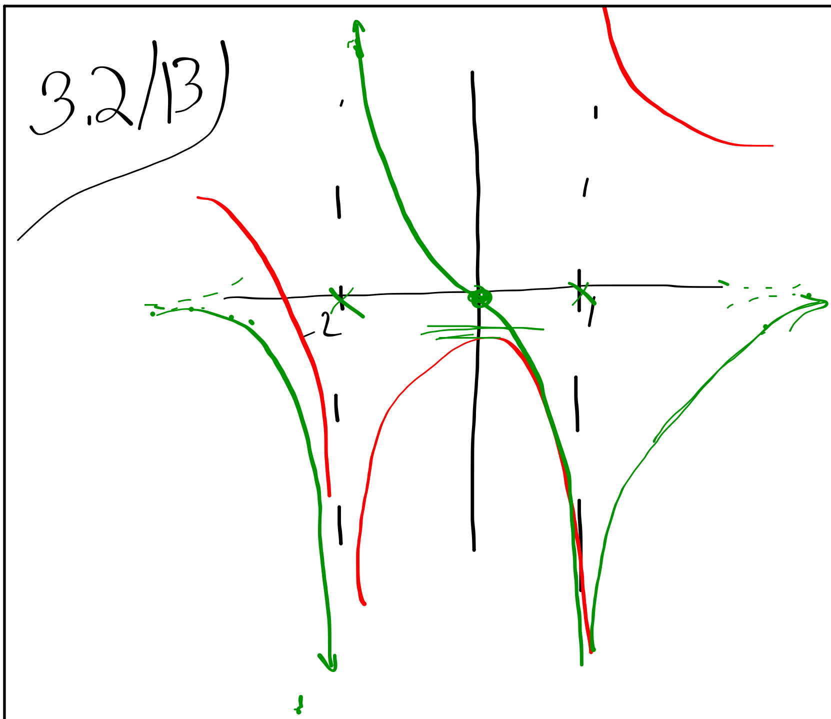
3.2/13)



day 19

$\frac{3.1}{20}$

Sketch graph of derivative

1) asymptotes ⇒ no deriv.

2) horizontal (-ish) give (0-ish) as a derivative

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3.1/20 for Sarah - with an h

day 19

$f(x) = \frac{1}{x}$, $P = (1, 1)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{\left(\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}\right)}{h} = \lim_{h \to 0} \frac{\left(\frac{-h}{x(x+h)}\right)\left(\frac{1}{h}\right)}{h\left(\frac{1}{h}\right)}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}\, x(x+h)}$$

$$= \lim_{h \to 0} \frac{-1}{x(x+h)} = \frac{-1}{x^2}$$

b) $f'(1) = \frac{-1}{(1)^2} = -1$

$P = (1, 1)$

$y - 1 = -(x - 1)$

$y = -x + 2$